# Exercises # 1: Review of probability theory

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September 4, 2019

We denote the combination numbers $C_n^p := \frac{n!}{p!(n-p)!} := \frac{n(n-1)\cdots(n-p+1)}{1\cdot 2\cdots p}$.

**Exercise 1** (Martin Gardner, *Scientific American* (1959))**.**

*(i) Mr. Jones has two children. The older child is a girl. What is the probability that both children are girls?*

*(ii) Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?*

**Exercise 2.** *What is the probability to obtain exactly 3 hearts when drawing 5 cards in a deck of 32 cards (containing exactly 8 hearts) . . .*

*(i) . . . simultaneously?*

*(ii) . . . successively without replacement?*

*(iii) . . . successively with replacement?*

**Exercise 3.** *An urn contains 4 white balls and 3 black balls. You draw 3 balls, one by one, without remise. What is the probability that the first ball is white, the second white and the third black?*

**Exercise 4.**

1. *State (and prove?) Bayes' formula.*
2. *Mr X has 100 dices among which 25 are* loaded *(unfair). For each piped dice, the probability to obtain a 6 is 0.5.*
   (a) *Mr X draws a randomly selected dice and obtains a 6. What is the probability that this dice is loaded?*
   (b) *Let $n \in \mathbb{N}^*$ be a positive integer. Mr X draws $n$ times a randomly selected dice and obtains a 6 each time. What is this time the probability $p_n$ that this dice is loaded?*
   (c) *Determine $\lim_{n\to\infty} p_n$. What does this mean?*