Exercises # 2: Random variables

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Please do at least 3 problems. Due in class on September 13, 2019. Python codes must be sent by email before September 13, 2019, 12:40pm.

Exercise 1. *A player shoots on a target with radius 10 inches, composed of concentric rings delimited by circles with radii 1, 2, . . . , 10 cm, and respectively numbered from 10 to 1. We suppose that the player reaches the the target all the time, and that the probability to hit ring number k is proportional to the area of this ring. Let X be the random variable which associates to each throw the number of the target.*

(1) What is the law (i.e. here p.m.f.) of X?

(2) The player wins $\$k$ if he hits ring number k for k between 1 and 6, but loses $\$2$ if he reaches the peripheral rings numbered 1 to 5. Is this game favorable to the player?

Hint: *(1) Compute the area of each ring. (2) Introduce the random variable equal to the gain of the player and compute its expectation...*

Exercise 2. *A restaurant has seventy guests every evening. The chef knows that, in average, two customers every five order a* crème brûlée. *He thinks that if he prepares thirty crème brûlées, then he will be able to satisfy all customers with seventy percent probability.*

(1) Is the chef right?

(2) How many crème brûlées (at least) does the chef need to bake to satisfy all customers with ninety percent probability?

Hint: *Introduce the random variable equal to the number of crème brûlées ordered on a given evening.*

Exercise 3. *Let X and Y be two Gaussian random variables (with values in $\mathbb{R}$, with respective means μ and ν, respective standard deviations σ and τ, and correlation ρ. What is $\mathbb{E}[X|Y = y], y \in \mathbb{R}$?*

Exercise 4 (Python)**.** *Write a function* `sample_discrete(n, q)` *that takes as input an integer* `n` *and a* pandas *series* `q`*, with length* `d = len(q)`*, index* `x = list(q.index)` *and values* `v = q.values`*, and returns a* numpy *array containing* `n` *independent samples from the discrete distribution which gives probability* `v[i]` *to the outcome* `x[i]`*,* `i` $= 1, 2, \ldots, d$.