Exercises # 5: Applications of Markov Chains

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Each problem is worth 10 points, points above 40 are bonus. Due at the beginning of class on October 18. Python codes must be sent by email before October 18, 12:40pm.

Exercise 1. *Exercise 63 page 286 in the textbook.*

Exercise 2. *Exercise 64 page 286 in the textbook.*

Exercise 3. *Exercise 66 page 286 in the textbook.*

Exercise 4. *Exercise 67 page 286 in the textbook.*

Exercise 5 (Python)**.** *First or all, simulate a "data" vector x containing 1,000 independent observations of Binomial distribution with $n = 2019$ and $p = 0.5$. This vector $x = (x_1, x_2, ..., x_{1,000})$ is now fixed.*

Now suppose that we have the following model for the data X: first we know that, given the value of a random variable N, $X_1, ..., X_{1,000}$ follow independent Binomial distribution with parameters $n = n$ and $p = 0.5$, i.e.

$$p(X = x|N = n) = \prod_{k=1}^{1,000} C_n^{x_k} \frac{1}{2^n}.$$

Additionally, we also know the general distribution of N: $\mathbb{P}(N = n) = \frac{6}{\pi^2 n^2}$.

Use the Metropolis Hasting algorithm to generate 10,000 samples $(Z_1, Z_2, ..., Z_{10,000}$ approximately following the conditional distribution $p(N|X = x)$, i.e. such that

$$p(Z_n = j) \approx p(N = j|X = x)$$

when $n \to \infty$, and plot their histogram. What is (an approximate value of) $\mathbb{E}[N|X = x]$? Note that the answer will depend on the values you sampled for x.

You can use numpy, scipy and matplotlib as well as all packages from the standard library.