Syllabus - STT 990 - Summer 2018

Stochastic processes, filtrations, and applications to finance

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1 Content and objectives

This course is a modern introduction the theory of continuous-time stochastic processes, with a leading application to modeling financial markets with asymmetric information.

After a brief review of probability, filtrations and conditioning, we will present fundamental tools and objects of stochastic analysis including Brownian motion, martingales, stochastic integration and stochastic differential equations, semimartingales and Girsanov's theorem. In an innovative approach, we will study the sensitivity of the results in the underlying information structure by considering expansions of the filtrations at various stage of the course. We will illustrate the theory with examples from mathematical finance such as option pricing, insider trading models, and if time allows, more advanced topics such as volatility modeling and estimation.

This course is intended for PhD students and advanced Master students in Statistics and in Mathematics. It will be most accessible to those with some background on basic measure-theoretic real analysis or probability theory, and will involve personal studies.

2 Class and office hours

Classes will be on Tuesdays & Thursdays, 4:10-6:55pm, C405 Wells Hall. Regular office hours will be on Tuesdays & Thursdays, 2-4 pm, C442 Wells Hall. Additional times can be arranged email by appointment.

3 Grading

There will be no exam. Short homework exercises will be given regularly; they will not be collected, but discussed at the beginning of class. The grade will be based on attendance and participation.

4 Course materials

There is no required textbook. The course will rely on the following classical references available from MSU library:

- Steven E. Shreve, *Stochastic calculus for finance II, Continuous-time models*, Springer (2004)
- Philip E. Protter, *Stochastic integration and differential equations*, Second edition, Springer (2005)
- Patrick Billingsley, *Probability and measure*, Third edition, Wiley (1995)

5 Course outline

I - Stochastic processes

1) Review of probability theory
2) Introduction to the Brownian motion and Poisson process
3) Existence of stochastic processes
   a) Kolmogorov extension theorem
   b) Construction of the Brownian motion

II - Itô's stochastic calculus

1) Martingales
2) Quadratic variation of the Brownian motion
3) Brownian integration
4) Itô's formula
5) Introduction to mathematical finance
   a) Binomial model
   b) Direct approach to the Black-Scholes paradigm
   c) Risk neutral pricing

III - Semimartingales

1) Complements on martingales
2) Local martingales
3) Semimartingales
4) Stochastic integration
5) Quadratic variation
6) Itô's formula
7) Mathematical finance with semimartingales

6 Tentative schedule

- Th. 05/17 : Course overview, review of probability theory and introduction to Brownian motion & Poisson process
- Tu. 05/22 : Existence of stochastic processes and
- Th. 05/24 : Construction of Brownian motion
- Tu. 05/29 : Quadratic variation of Brownian motion, Brownian stochastic integral and Itô's formula
- Th. 05/31 : Mathematical finance, binomial model and Black-Scholes paradigm
- Tu. 06/05 : No-arbitrage pricing and Girsanov theorem
- Th. 06/07 : Local martingales and semimartingales
- Tu. 06/12 : Semimartingale stochastic integration
- Th. 06/14 : Quadratic variation of semimartingales and Itô's formula
- Tu. 06/19 : Decomposition of semimartingales
- Th. 06/21 : Mathematical finance with semimartingales
- Tu. 06/26 : Expansions of filtrations and asymmetric information
- Th. 06/28 : Levy processes, volatility estimation, complements on SDEs, or another topic, according to progress and interest